Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

upper or the lower half, preferably of the middle columns; but as we have seen in the 10-square, several other points may be found suitable for the exchange.

This plan and process for developing to so high a degree of excellence, the oddly-even squares, starting with the 2-square, and constantly employing its endless combinations, is equally applicable to the evenly-even squares. They do not need it, as there are many well-known, convenient and expeditious methods for their construction. However, in closing we will give a specimen of the 4-square, type of all that class, showing the pervading influence therein of the truly ubiquitous 2-square.

1	5	13	9
13	9	1	5
1	5	13	9
13	9	1	5

Fig. 13.

0	3	0	3
1	2	1	2
3	0	3	0
2	1	2	1

Fig. 14.

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

Fig. 15.

The primaries Figs. 13 and 14 as well as the complete square Fig. 15, singly and together fill the bill with no discount. Each is a quartered square, magic to a high degree. Each contains numerous 2-squares, four being compact in the quarters and five others overlapping. And there are many more variously scattered abroad especially in Fig. 14. While these specimens seem to conform exactly to foregoing rules they were actually made by continuous process using the knight's move 2:1 and 1:2. The pattern is rhomboidal.

In all the combinations here presented, and especially in these last specimens, the 2-square is pervasive and organic. "So we have a symmetry," as one of our philosophical writers has said—"which is astonishing, and might be deemed magical, if it were not a matter of intrinsic necessity."

D. F. SAVAGE.

NOTES ON ODDLY-EVEN MAGIC SQUARES.

The article on oddly-even squares by Mr. D. F. Savage in the current number of *The Monist* is a valuable contribution to the general literature on magic squares. Mr. Savage has not only clearly described a clever and unique method of constructing oddly-even squares, but he has also lucidly demonstrated the apparent limit of their possible perfection.

The arrangement of concentric quartets of four consecutive numbers in his 6×6 square is strikingly peculiar, and in studying this feature it occurred to the writer that it might be employed in the development of these squares by a direct and continuous process, using the arithmetical series 1 to n^2 taken in groups of four consecutive terms, 1.2.3.4.—5.6.7.8. etc.

The constructive method used by Mr. Savage is based on the well-known and elegant plan of De la Hire, but the two number series which he has chosen for the first and second auxiliary squares are unusual, if not entirely new. It is difficult to see how these unique squares could have been originally evolved by any other method than that adopted by Mr. Savage, and the different constructive scheme presented herewith must be regarded as only a natural outcome of the study of his original plan. It may also tend to throw a little additional light on the "ubiquitous 2×2 square" and to make somewhat clearer the peculiar features that obtain in these oddly-even squares.

Referring to Fig. 1 (which is a reflected inversion of Fig. 7 in the article by Mr. Savage and therefore requires no further explanation) it will be seen that this square contains nine quadrate groups of the series 0.1.2.3., the numbers in each group being scattered in each of the 3×3 quarters, and in concentric relationship to the 6×6 square. The numbers of these quadrate groups are not, however, distributed in any apparent order as viewed numerically, although the diagram of their constructive forms, which will be referred to later on, reveals the symmetry of their arrangement.

3	0	1	3	1	1
0	3	2	0	2	2
3	0	1	3	1	1
0	2	2	0	3	2
3	1	1	3	0	1
0	2	2	0	3	2

Fig. 1.

Any middle outside cell of the 3×3 quarters containing a cypher can be used as a starting point for a 6×6 square, and inspection will show four such cells in Fig. 1.

Selecting the second cell from the left in the upper line to start from, the numbers in the quadrate concentric group of which this cell is a member will be seen to have the formation shown in Fig. 2A, so the first group of four numbers (1.2.3.4) in the series 1 to 36 are similarly placed in Fig. 3, running also in the same *relative* numerical order.

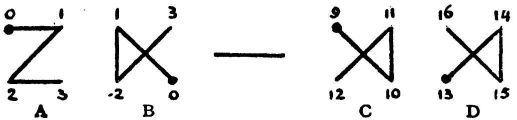

Fig. 2.

To secure magic results in the completed square, each succeeding entry in the 3×3 quarters must follow the last entry in *magic square order.* For the next entry in Fig. 3 there is consequently a choice of two cells. Selecting the lower right-hand corner cell of the 3×3 quarter of Fig. 1 used at the start, it is seen to be occupied by 1, and the formation of the quadrate concentric group is as shown in Fig. 2B. The terms 5.6.7.8. are therefore entered in Fig. 3 in similarly located cells, and as before, in the same relative numerical order. The next quadrate group of 9.10. 11.12 have the order shown in Fig. 2C,—13.14.15.16 are arranged as in Fig. 2D, and so on until all of the 36 cells are filled. The resulting finished square is shown in Fig. 4.

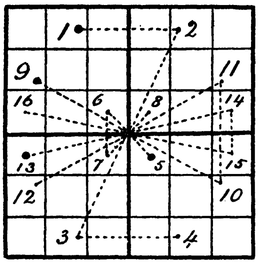

Fig. 3.

32	1	22	24	2	30
9	20	27	25	19	11
16	33	6	8	34	14
13	35	7	5	36	15
12	18	26	28	17	10
29	3	23	21	4	31

Fig. 4.

Fig. 5 shows the different forms of the nine consecutive quadrate groups contained in Fig. 4, written in regular order, and it discloses the harmonious relationship of the couplets.

There are two alternative forms for the first group, as shown in Fig. 5. If the square is to be pan-diagonal or continuous at the expense of the summation of two vertical columns, the right-hand form must be used, but if the square is to be strictly magic at the expense of making four diagonals incorrect, then the left-hand form is correct.

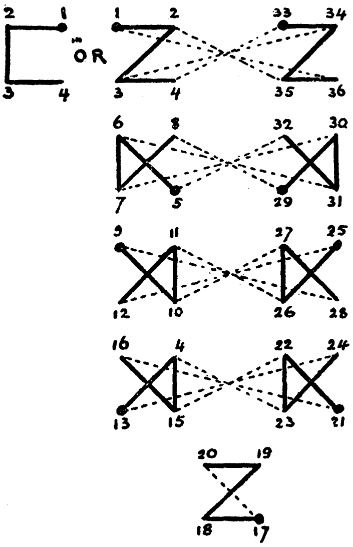

Fig. 5.

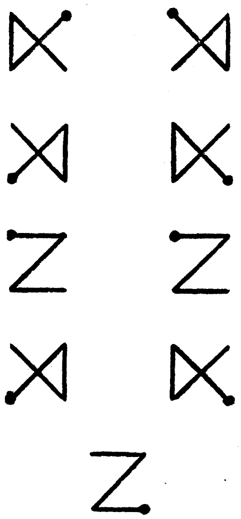

Fig. 6.

This graphic presentation of number order is instructive, as it shows at a glance certain structural peculiarities which are not apparent on the face of the square.

8	25	22	24	26	6
33	20	3	1	19	35
16	9	30	32	10	14
13	11	31	29	12	15
36	18	2	4	17	34
5	27	23	21	28	7

Fig. 7.

Another of the many variants of this 6×6 square may be made by starting from the fourth cell of the second line in Fig. 1, this being also a middle outside cell of a 3×3 square.

Under this change the forms of the quadrate groups are shown in Fig. 6, the resulting square being given in Fig. 7.

When these 6×6 squares are made pan-diagonal, i. e., perfect in all their diagonals, the normal couplets are arranged in harmonic relation throughout the square, the two paired numbers that equal n^2+1 being always located in the same diagonal and equally spaced $n/2$ cells apart. If the square is made strictly magic, however, this harmonic arrangement of the couplets is naturally disturbed in the imperfect diagonals.

The above remarks and rules will of course apply generally to 10×10 and larger squares of this class. A 10×10 square modified from Mr. Savage's example to secure the harmonic arrangement of the couplets, as above referred to, is given in Fig. 8.

20	41	86	32	73	75	31	85	43	19
89	36	79	1	48	46	2	80	34	90
63	5	50	93	40	39	95	49	7	64
56	97	23	68	9	10	66	24	98	54
25	72	14	59	84	83	57	13	71	27
26	70	16	58	82	81	60	15	69	28
55	99	21	67	11	12	65	22	100	53
62	6	52	94	37	38	96	51	8	61
91	35	77	3	47	45	4	78	33	92
18	44	88	30	74	76	29	87	42	17

+1 —1

Fig. 8.

W. S. Andrews.

Schenectady, N. Y.

BOLYAI, LOBATCHEVSKY, RUSSELL.

In his book on the *Foundations of Mathematics,* Dr. Carus wonders what sort of pistols Bolyai used in the famous duels where, like Ivanhoe, he discomfited the whole series of challengers. But Bolyai was a compound of Saladin and Richard, fighting with a Damascus blade which cut silken cushions or chopped iron. Franz Schmidt told me in Budapesth that his father had seen Bolyai lop off a spike driven into his doorpost, and that some of his duels were to the death.

Bolyai published only 24 pages of text, but this is the most extraordinary two dozen pages in the history of human thought. The very first page would kill or cure my genial friend Mr. Russell.